MATH 142 Sample Final Exam Problems

Question 1 Evaluate the following integrals:

a) $\displaystyle\int \frac{\cos(5-\ln x)}{x}\,dx$ b) $\displaystyle\int \frac{dt}{(\tan t)\sqrt{4-\sin^2 t}}$

c) $\displaystyle\int_{-2}^{1} \frac{dx}{\sqrt{x^2+4x+11}}$ d) $\displaystyle\int \frac{x^5-x^4-3x+5}{x^4-2x^3+2x^2-2x+1}\,dx$ Hint: Denom factors as $(x^2+1)(x-1)^2$

e) $\displaystyle\int \arcsin\sqrt{x}\,dx$ f) $\displaystyle\int_0^{\pi/6} x\tan^2(2x)\,dx$

g) $\displaystyle\int \frac{d\theta}{\sqrt{1+\sqrt{\theta}}}$

Question 2 Find the equation of the tangent line to the curve defined implicitly by the equation $y\log_3 y = 5^{2x}-1$ at the point $(0,1)$.

Question 3 Determine the following limits:

a) $\displaystyle\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln x}$ b) $\displaystyle\lim_{x\to e^+} (\ln x)^{1/(x-e)}$

c) $\displaystyle\lim_{x\to\infty} \sqrt{x^2+100x}-\sqrt{x^2+50x}$ d) $\displaystyle\lim_{x\to 0^+}\left(\frac{3x+1}{x}-\frac{1}{\sin x}\right)$

Question 4

a) Find the length of the curve $y=\dfrac{1}{6}x^3+\dfrac{1}{2x}$, $1\le x\le 2$.

b) Find the area of the surface obtained by rotating this curve about the x–axis.

Question 5 Let $f(x)=6x-\arctan(4x)$.

a) Show that $f(x)$ is one-to-one on the interval $(-\pi/8,\pi/8)$.

b) Find the derivative $(f^{-1})'(0)$.

Question 6 Find the volume of the solid of revolution obtained by rotating the curve $y=\dfrac{\ln x}{\sqrt{x}}$, $1\le x\le e^4$, about the line $x=-2$.

Question 7 Find the centroid ($\rho=1$) of the region bounded by the parabolas $y=2x^2-4x$ and $y=2x-x^2$.

Question 8 A differential equation problem (population growth, radioactive decay, interest, cooling, etc.).

Question 9 A work problem about pumping water out of a tank.

The topics covered on the final are Chapter 5.5, Chapter 6, Chapter 7 (except 7.6), Chapter 8, Chapter 9.1–9.3, Chapter 10.1 and 10.4.